STRESS STATE OF AN ORTHOTROPIC PLATE AT THE CURVILINEAR HOLES

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The solutions of the plane problems about stress concentration near curvilinear holes in an orthotropic plane with smooth and piece-smooth contours are constructed using the method of singular integral equations. Numerical results for orthotropic plates with holes of various shapes under uniaxial symmetric loading at infinity are obtained. The distributions of contour normal stresses for a narrow slot and an elliptical hole in orthotropic and quasi-orthotropic planes with the same ratio of the main elastic module of the material are compared.

Keywords: orthotropic plate, curvilinear holes, stress concentration, singular integral equations

Introduction Plane problems of stress concentration near holes were studied by various methods. The method of conformal reflections [1–4], which, in combination with the series method, was also used for a multi-connected anisotropic region [5], was effective for one curved hole in the isotropic plane. Analytical and numerical methods are used to study the stress-strain state of elastic anisotropic plates. One of the most effective methods for a multi-connected flat elastic region with holes and cracks turned out to be the method of singular integral equations (SIE). Closed analytical solutions for some partial problems, which in particular include the determination of the stress concentration in an orthotropic plane with elliptical, triangular and rhombic holes with rounded vertices were obtained.

Integral Equation for an Anisotropic Plane with a Curvilinear Hole. Let us assume that self-balanced stresses are set on the closed contour L of the curvilinear hole

$$N + iT = p(t),\ t \in L\ .$$

The main vector (X, Y are it projections on the x, y axis , respectively) and the main moment M of the external load are equal to zero:

$$X + iY = -i\int_L p(t)dt = 0,\quad M = -\mathrm{Re}\int_L \bar{t}p(t)dt = 0\ . \tag{1}$$

Let us take the complex stress potentials in the form

$$\Phi_1(z_1)=\frac{1}{2\pi i}\int\limits_{L_1}\frac{\phi_1'(\tau_1)}{\tau_1-z_1}d\tau_1,\quad \Phi_2(z_2)=\frac{1}{2\pi i}\int\limits_{L_2}\frac{\phi_2'(\tau_2)}{\tau_2-z_2}d\tau_2 .$$

The problem is reduced to solving SIE [9]

$$\frac{1}{\pi}\int\limits_{L_1}[K_1(\tau_1,t_1)\phi_1'(\tau_1)d\tau_1+L_1(\tau_1,t_1)\overline{\phi_1'(\tau_1)}d\overline{\tau}_1]=P_1(t_1),\ t_1\in L_1 , \tag{2}$$

kernels $K_1(\tau_1,t_1)$, $L_1(\tau_1,t_1)$ and the right side $P_1(t_1)$ of which are

$$\begin{aligned} K_1(\tau_1,t_1)&=\frac{(\mu_1-\mu_2)}{2}\left(\frac{1}{\tau_1-t_1}\frac{dt_1}{dt}+\frac{1}{\overline{\tau}_2-\overline{t}_2}\frac{d\overline{t}_2}{dt}\right),\\ L_1(\tau_1,t_1)&=-\frac{(\overline{\mu}_1-\mu_2)}{2}\left(\frac{1}{\overline{\tau}_1-\overline{t}_1}\frac{d\overline{t}_1}{dt}-\frac{1}{\overline{\tau}_2-\overline{t}_2}\frac{d\overline{t}_2}{dt}\right), \end{aligned} \tag{3}$$

$$P_1(t_1)=\frac{1}{2}\left[(1-i\mu_2)p(t)-(1+i\mu_2)\overline{p(t)}\frac{d\overline{t}}{dt}\right] . \tag{4}$$

For orthotropic body expressions (2) and (3) take the form. Integral Eq. (4) has single solution under additional conditions

$$\int\limits_{L_1}P_1(t_1)dt_1=0,\quad \int\limits_{L_1}\overline{t}_2P_1(t_1)dt_1=0 , \tag{5}$$

which meet the equilibrium conditions (1).

Let us modify Eq. (2) so as to obtain a single solution for an arbitrary right-hand side. Let us take advantage of the approach previously used in similar problems for an isotropic body. Let us add to the left-hand part of Eq. (2) certain functionals, which ensure the uniqueness of the solution, but are equal to zero if conditions (5) are met. Such a modified equation has the form

$$\int\limits_{L_1}[K_1(\tau_1,t_1)\phi_1'(\tau_1)d\tau_1+L_1(\tau_1,t_1)\overline{\phi_1'(\tau_1)}d\overline{\tau}_1]-\frac{1}{2i}\frac{M}{(\overline{t}_1-\overline{z}_1^0)^2}+\frac{a_1^0}{l}\frac{ds_1}{dt_1}=P_1(t_1),$$

where z_1^0 is an arbitrary point, corresponding to the region S^+ occupied by the hole in the plane z, of the region S_1^+ in the plane z_1; l is an arbitrary length dimension parameter; s_1 is an arc abscissa on the contour L_1 corresponding to the point t_1;

$$a_1^0 = \int\limits_{L_1} \phi_1'\left(t_1\right) dt_1; \; M = i \int\limits_{L_1} [\overline{t}_2 \phi'(t_1) dt_1 - t_2 \overline{\phi_1'(t_1)} d\overline{t}_1] .$$

From the known formula for the main vector of the applied load

$$X + iY = 2\,\mathrm{Re}[\mu_1 \int\limits_{L_1} \Phi_1(t_1) dt_1 + \mu_2 \int\limits_{L_2} \Phi_2(t_2) dt_2] - 2i\,\mathrm{Re}[\int\limits_{L_1} \Phi_1(t_1) dt_1 + \int\limits_{L_2} \Phi_2(t_2) dt_2]$$

the relation $a_1^0 = \int\limits_{L_1} \phi_1'\left(t_1\right) dt_1 = \tilde{A}X + \tilde{B}Y$, where $\tilde{A}, \tilde{B}$ are the complex constants expressed in terms of elastic parameters μ_1, μ_2 is received. If the main vector is equal to zero than constant $a_1^0 = 0$.

Numerical Results. On the basis of the numerical solution of SIE (12) on a free closed contour we obtained the distribution of normal stresses along the contour of smooth holes in the orthotropic

$$\sigma_S = \sigma_{xx} + \sigma_{yy} = 2\,\mathrm{Re}\{(1-\gamma_1^2)\Phi_1(z_1) + (1-\gamma_2^2)\Phi_2(z_2)\}$$

and quasi-orthotropic ($\gamma = \sqrt[4]{E_x/E_y} = 1,3716$) [9]

$$\sigma_s = \mathrm{Re}\left\{2(1+\gamma^2)\Phi_1(z_1) + (1-\gamma^2)\left[\overline{z}_1 \Phi_1'(z_1) + \Psi_1(z_1)\right]\right\}$$

plates under their uniaxial tension along the axis Oy($\sigma_y^\infty = p$). We considered the shape of holes in the form of an ellipse (Fig. 1*a*) or a physical slot (Fig. 1*b*) with the same relative radii of rounding ($\varepsilon = \rho/l = 0,2$) at the tips for parameters $\gamma_1 = 2,3337$; $\gamma_2 = 0,6594$ for ETF material [13]. Here $\Phi_1(z_1)$ and $\Psi_1(z_1)$ are complex stress potentials in the quasiorthotropic plane.

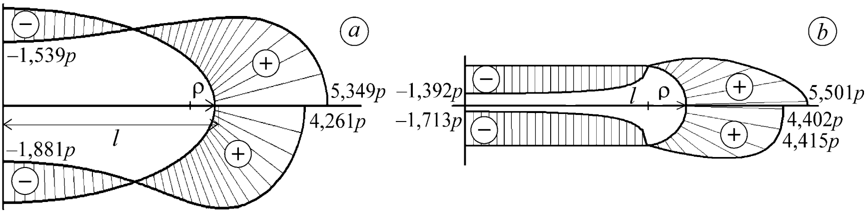

Fig. 1. Stress distribution σ_S along the contour of (a) an elliptical hole and (b) narrow slot in orthotropic (upper half-plane) and quasiorthotropic (lower half-plane) planes with the same ratios between the modulus of elasticity of materials E_x / E_y under uniaxial tension $\sigma_y^{\infty} = p$.

Thus, the stress distributions along the contours of curved holes in orthotropic and quasiorthotropic planes with the same ratios of the elastic modules of these materials under a given load are qualitatively close (Fig. 1). However, for both forms of holes, the maximum tensile stresses in rounded tips are always greater in an orthotropic plate and compressive stresses in a quasiorthotropic plate.

Conclusions. The distribution of stresses near the sharp and rounded tips of holes in orthotropic plates was investigated using the SIE method. An asymptotic approach was used to find the SIF in the sharp angular tips of the hole contour, when SIF is found on the basis of the SCF in the corresponding rounded tips. Numerical results were obtained for uniaxial tension at infinity of orthotropic plates with curvilinear holes of various shapes. The distributions of normal stresses along the contour of the physical slot and elliptical hole in orthotropic and quasiorthotropic plates with the same ratio of the main elastic modules of the materials were compared. It was established that the relative difference between these stresses for the considered materials is small (up to 25%).

1. *Kaminskyi A.A.* Brittle fracture near holes. – Kyiv Naukova Dumka, 1982. (in Russian)
2. *Muskcelishbili N.I.* Some basic problems of the mathematical theory of elasticity. – Moscow: Nauka, 1966. (in Russian)
3. *Savin G.N.* Stress concentration near holes. – Moscow: Leningrad, 1951. (in Russian)
4. *Savin G.N.* Sress distribution near holes. – Moscow: Nauka, 1968. (in Russian)
5. *Kaloerov S.A. and Petrenko A.V.* Two-dimensional problems of electromagnetic-elasticity for multy connected bodies. – Donetsk: Yugo-Vostok, 2011. (in Russian)