ACOUSTIC PLANE WAVE SCATTERING FROM A TRUNCATED SEMI-INFINITE SOFT CONE AT AXIAL INCIDENCE

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The axially symmetric diffraction problem of the plane acoustic wave from the truncated semi-infinite soft cone is considered. The problem is solved rigorously using the analytical regularization procedure. The obtained solution is used to study the cone cut on scattering features.

Keywords: acoustic wave, truncated cone, analytical regularization procedure

An essential component in developing effective methods of acoustic non-destructive testing is solving model problems of diffraction. This solution helps understand the phenomena occurring in the material during testing and find optimal equipment parameters and settings for specific control tasks. Also, it is important for identifying and sizing defects in material, such as cracks or inclusions. The studying diffraction problem from a cone is significant since it is widely used in theoretical physics and is suitable for modelling defects. Accordingly, we consider the diffraction problem of the plane acoustic wave $U^{(i)} = \exp(ikr\cos\theta)$ from the truncated semi-infinite soft cone $Q:\{r \in (c_1, \infty)$, $\theta = \gamma$, $\varphi \in [0, 2\pi)\}$ in axial incidence and reduce it to the set of series equations

$$\sum_{\substack{n=1\\ -1\le\eta\le 1}}^{\infty} \bar{x}_n P_{n-1}(\eta) = \begin{cases} \displaystyle\sum_{p=1}^{\infty} y_p^{(1)} P_{\nu_p - 1/2}(\eta); \\ \qquad \chi < \eta \le 1 \\ \displaystyle\sum_{k=1}^{\infty} P_{\mu_k - 1/2}(-\eta)\left[y_k^{(2)} + A_k I_{\mu_k}(\rho_1)\right]; \\ \qquad -1 \le \eta < \chi \end{cases} \tag{1}$$

$$\sum_{\substack{n=1\\ -1\le\eta\le 1}}^{\infty} \bar{x}_n P_{n-1}(\eta) \frac{I'_{z_n}(\rho_1)}{I_{z_n}(\rho_1)} = \begin{cases} \displaystyle\sum_{p=1}^{\infty} y_p^{(1)} P_{\nu_p - 1/2}(\eta) \frac{K'_{\nu_p}(\rho_1)}{K_{\nu_p}(\rho_1)}; \\ \qquad \chi < \eta \le 1 \\ \displaystyle\sum_{k=1}^{\infty} P_{\mu_k - 1/2}(-\eta)\left[y_k^{(2)} \frac{K'_{\mu_k}(\rho_1)}{K_{\mu_k}(\rho_1)} + A_k I'_{\mu_k}(\rho_1)\right]. \\ \qquad -1 \le \eta < \chi \end{cases} \tag{2}$$

Here $\bar{x}_n$, $y_p^{(1)}$, $y_k^{(2)}$ are the unknown coefficients; $\rho_1 = sc_1$ ($s = -ik$, k is the wave number), c_1 is the truncation radius; $\eta = \cos\theta$; $\chi = \cos\gamma$, γ is the cone-generating angle; $z_n = n - 1/2$; $K_\alpha(\cdot)$, $I_\alpha(\cdot)$ are the Macdonald function and the Bessel function, respectively; $P_{\alpha-1/2}(\cdot)$ is the Legendre function; $A_k = \sqrt{2\pi^3}\mu_k P_{\mu_k-1/2}(\chi)/\cos\pi\mu_k\partial_\mu P_{\mu_k-1/2}(-\chi)$; ν_p and μ_k are positive roots of transcendent equations as

$$P_{\nu_p-1/2}(\chi) = 0,\ P_{\mu_k-1/2}(-\chi) = 0.$$

Using the orthogonality properties of Legendre functions and the method of analytic regularization [1], we reduce the problem of determining the unknown coefficients in (1), (2) to the solution of an infinite system of linear algebraic equations (ISLAE) of the second kind, which we write as

$$\mathrm{X} = \mathrm{A}^{-1}(\mathrm{A} - \mathrm{A}_{11})\mathrm{X} + \mathrm{A}^{-1}\mathrm{F}. \tag{3}$$

Here $\mathrm{X} = \{x_n\}_{n=1}^{\infty}$ is the unknown vector, $x_n = \bar{x}_n P_{z_n-1/2}(\chi)$; $\mathrm{F} = \{f_q\}_{q=1}^{\infty}$ is the known vector; $\mathrm{A}_{11} = \{a_{qn}^{(11)}\}_{q,n=1}^{\infty}$ is the infinite matrix; A, A^{-1} are the regularization operators defined in [2].

The ISLAE solution (3) exists in the class of sequences $x_n = O(n^{-3/2})$, ensures the fulfilment of all necessary conditions, and can be obtained by the reduction method with a given accuracy [1].

Based on the solution (3), we consider the scattering features of the truncated semi-infinite soft cone in the far-field zone. The reliability of the obtained results is defined by comparing them with the known in limiting cases. The influence of cone cut on characteristics of the diffracted field in a wide frequency range is studied. The possibility of focusing the radiation field in the direction of wave propagation and the influence of this effect on the structure parameters is established.

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