THE APPLICATION OF A BICONICAL SCATTERER FOR MODELING OF THE PLANE SURFACE DEFECT IN THE PRESENCE OF THE CONICAL PROBE

OLEKSIY SHARABURA

Karpenko Physico-Mechanical Institute of NAS of Ukraine

The solution of the wave diffraction problem that is obtained in [1] is modified and applied for modelling of the axially symmetric transverse magnetic (TM) waves scattering from the conical probe in the presence of half-spherical defect on the plane surface. This structure is modelled by a bicone with semi-infinite and truncated semi-infinite perfectly conducting conical shoulders and the spherical diaphragm. The structure is excited by the radial electric dipole. The mode-matching and analytical regularization techniques are used for the solution of the problem. Based on the obtained solution, the near and the far fields scattered from the defect are analysed for determining the information parameters for its identification.

Keywords: bicone, spherical diaphragma, analytical regularization

Introduction. Development of methods for diagnostics of materials and structures requires the study of regularities of electromagnetic/acoustic waves interaction with the defects and the structural elements. The use of the canonical models for this purpose is important to simplify the analysis. In [1] we have studied the wave diffraction from the bicone which is formed by the perfectly conducting semi-infinite and truncated semi-infinite conical shoulder. The truncated shoulder includes the internal spherical diaphragm which forms an open-ended cavity. This structure has been considered in spherical coordinate system and excited axially symmetric by the TM-wave that is produced by the radial electric dipole. Here, we study the scattering properties of this structure, when one of the shoulders degenerate into the plane and forms the cavity-like defect in the presence of the semi-infinite conical probe.

Fig. 1. Geometry of the problem.
**Problem solution.** Let us consider, in a spherical coordinate system \((r, \theta, \varphi)\), the perfectly conducting biconical scatterer (see Fig. 1). We formulate the mixed boundary value problem to determine \(H_\varphi\) - field diffracted from biconical scatterer as

\[
\nabla^2 H_\varphi - (r \sin \theta)^{-2} H_\varphi + k^2 H_\varphi = 0, \\
(sin \theta)^{-1} \partial_r \left[ \sin \theta (H_\varphi + H_\varphi^i) \right] = 0,
\]

if \(r, \theta \in \{(r, \theta) \mid r \in (0, \infty), \theta = \gamma_1 \} \cup \{r \in (a_1, \infty), \theta = \pi / 2 - 0 \} \cup \{r \in (a_1, c_1), \theta = \pi / 2 + 0 \},

\]

\[
r^{-1} \partial_r [r(H_\varphi + H_\varphi^i)] = 0, \text{ if } r, \theta \in \{r = c_1, \theta \in [\gamma_2, \pi / 2] \}.
\]

Here, \(k\) is the wave number. We apply the technique devoped in [1] for solution of this problem in the limited case when \(\gamma_2 = \pi / 2\). The time factor \(e^{-\mathrm{i}t}\) is suppressed throughout this paper. Finally, the boundary value problem is reduced to the infinite system of linear algebraic equations of the second kind as

\[
X = A_1^{-1}(A_1 - A_{11})X - A_1^{-1}B_{11}X + A_1^{-1}F. 
\]

Here, \(X: \{x_n\}_{n=1}^{\infty}\) is the unknown vector, \(x_n\) is the unknown complex amplitude of the sub-region mode; \(A_{11} : \{a_{qn}^{(1,1)}\}_{q,n=1}^{\infty}\) and \(B_{11} : \{b_{qn}^{(1,1)}\}_{q,n=1}^{\infty}\) are infinite matrices with elements

\[
a_{qn}^{(1,1)} = \frac{sa_q W[K_{\xi_q} I_{z_n}(sa_i)I_{z_n}(sa_j)]}{(\xi_q^2 - z_n^2)K_{\xi_q}(sa_i)I_{z_n}(sa_j)}, \\
b_{qn}^{(1,1)} = \begin{cases} s_a W[I_{\xi_q} I_{z_n}(sa_i) Y_{\xi_q} (\omega)] & \xi_q \in \{\nu_k\}_{k=1}^{\infty}, \\
(\xi_q^2 - z_n^2)I_{\xi_q}(sa_i)I_{z_n}(sa_j) & \xi_q \in \{\mu_p\}_{p=1}^{\infty}. \end{cases}
\]

\(Y_{\nu}(\omega)\) is the known function, \(K_{\nu}(\cdot), I_{\nu}(\cdot)\) are Macdonald and modified Bessel functions; \(\xi_q, z_n\) are known indexes; \(A_1, A_1^{-1}\) are regularizing operators. The diffracted field characteristics are studied based on the numerical solution of the equation (3).

**Conclusions.** The solution of the wave diffraction problem from the biconical scatterer in the limited case when one of the shoulders degenerates into the plane with the semi-spherical cavity is obtained. This solution is applied for the analysis of the far and near fields scattering from the cavity-like defect. The resonance regimes of the cavity excitation are studied for detection of the defect.