ACOUSTIC PLANE WAVE SCATTERING FROM A TRUNCATED SEMI-INFINITE SOFT CONE WITH AN INTERNAL SPHERICAL-CAP TERMINATION

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The axially symmetric diffraction problem of a plane acoustic wave from the truncated semi-infinite soft cone with an internal spherical-cap termination is examined. The problem is rigorously solved using the analytical regularization procedure. The resulting solution is used to study the scattering characteristics as a function of the geometrical parameters of the structure.

Keywords: truncated cone, sphere-conical cavity, analytical regularization procedure

The interest in studying the class of diffraction problems characterized by sphero-conical geometry arises from its potential wide applications in modelling various problems in applied physics and engineering. Depending on the geometric parameters, this cavity can be considered as the active component of waveguide probes used in microwave diagnostics, concave spherical reflectors, antennas, and as a model of an under-surface defect. In the scientific literature, sphero-conical cavities are well explored in the electromagnetic context [1]. To enhance understanding of the scattering features of a sphero-conical cavity from an acoustic perspective, the axially symmetric diffraction problem of a plane acoustic wave incident on the truncated soft cone with an internal spherical-cap termination

$$\mathrm{Q}=\left\{(r,\theta,\varphi)\,|\,r\in\begin{Bmatrix}(c_1,\infty),\ \theta=\gamma+0\\(c_1,c),\ \ \theta=\gamma-0\end{Bmatrix},\theta=\gamma\right\}\bigcup_{\varphi\in[0,2\pi)}\quad \theta,\varphi)\,|\,r=c,\theta\in[0,\gamma]\}$$

is considered. The diffraction problem is reduced to an infinite system of linear algebraic equations (ISLAE) of the second kind using the analytical regularization procedure [2,3] as follows:

$$\mathrm{X}=\mathrm{A}^{-1}(\mathrm{A}-\mathrm{A}_{11})\mathrm{X}-\mathrm{A}^{-1}\mathrm{B}\mathrm{X}+\mathrm{A}^{-1}\mathrm{F}\,. \tag{1}$$

Here, $\mathrm{X}=\{x_n\}_{n=1}^{\infty}$ is the unknown vector; $\mathrm{A}_{11}:\{a_{qn}^{(11)}\}_{q,n=1}^{\infty}$, $\mathrm{B}:\{b_{qn}\}_{q,n=1}^{\infty}$ are infinite matrix operators

$$a_{qn}^{(11)}=\frac{\rho_1 W[K_{\xi_q} I_{z_n}]\rho_1}{(\xi_q^2-z_n^2)K_{\xi_q}(\rho_1)I_{z_n}(\rho_1)},$$

$$b_{qn}=\begin{cases}-\dfrac{\rho_1 W[I_{\xi_q} I_{z_n}]\rho_1}{(\xi_q^2-z_n^2)I_{\xi_q}(\rho_1)I_{z_n}(\rho_1)}\left(\dfrac{K_{\xi_q}(\rho_c)}{K_{\xi_q}(\rho_1)}\dfrac{I_{\xi_q}(\rho_1)}{I_{\xi_q}(\rho_c)}\right), \xi_q\in\{\nu_p\}_{p=1}^{\infty},\\ 0,\ \xi_q\in\{\mu_k\}_{k=1}^{\infty},\end{cases}$$

where $\rho_1=sc_1$, $\rho=sc$ ($s=-\mathrm{i}k$, k is the wave number), c_1 is the radial coordinate of the rib of the hole, c is the radius of the spherical diaphragm, $c>c_1$; γ is the spherical aperture angle; $K_\eta(\cdot)$, $I_\eta(\cdot)$ are the Macdonald function and the Bessel function, respectively; $W[\cdot]$ is the Wronskian, $W[\alpha\beta]_o=\beta'(o)\alpha(o)-\beta(o)\alpha'(o)$; z_n, ξ_q are respectively positive zeroes and poles of an even meromorphic function

$$M(\nu,\gamma)=\pi^{-1}\cos(\pi\nu)\left\{P_{\nu-1/2}(\cos\gamma)P_{\nu-1/2}(-\cos\gamma)\right\}^{-1},$$

which is regular in the strip $\Pi:\{|\operatorname{Re}\nu|<1/2\}$, does not have zeroes in Π and $M(\nu,\gamma)=O(\nu)$ at $|\nu|\to\infty$; $\mathrm{F}=\{f_q\}_{q=1}^{\infty}$ is the known vector; A, A^{-1} are the regularization operators defined in [2].

The ISLAE solution (1) exists in the class of sequences $x_n=O(n^{-3/2})$ that ensures the fulfilment of all necessary conditions, and can be obtained by the reduction method with a given accuracy [2,3]. Based on solution (1), the scattering characteristics of the structure Q are analysed.

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