

**MATHEMATICAL MODELING OF  
ELECTROCARDIOGRAPHIC SIGNALS USING TEMPORAL  
RHYTHM AND AMPLITUDE VARIABILITY FUNCTIONS**

LYUBOMYR MOSIY<sup>1</sup>, ANDRIY SVERSTIUK<sup>1,2</sup>

1. Ternopil Ivan Puluj National Technical University, Ukraine  
2. I. Horbachevsky Ternopil National Medical University, Ukraine

Presents discrete mathematical models for electrocardiographic signal (ECS) analysis based on temporal rhythm functions and amplitude variability of characteristic waves P, Q, R, S, T. The proposed approach combines cyclic discrete random process theory with amplitude-temporal characteristics to enable comprehensive assessment of cardiac signals. Experimental validation on patients with conditional norm and extrasystole demonstrates high sensitivity to pathological changes, with variance increasing by 5–6 orders of magnitude for amplitude variability in pathological cases.

*Keywords:* ECS modeling, amplitude variability, temporal rhythm function, cyclic discrete random process, cardiac diagnostics

**Introduction.** Cardiovascular diseases remain the leading cause of mortality worldwide, necessitating effective methods for early diagnosis. While electrocardiography provides non-invasive access to cardiac electrical activity, existing automated analysis systems predominantly focus on temporal parameters, with insufficient attention to amplitude variability dynamics [1, 2]. The analysis of ECS processing and classification methods can be significantly improved by applying cyclic discrete random processes (CDRP) [3]. The use of statistical heart rate indicators as input data allows for a more targeted analysis and reduces the requirements for the size of the data set, which facilitates the detection of critical time events and improves the efficiency of the predictive model [4].

This study develops a mathematical model of ECS as a CDRP with integrated temporal rhythm and amplitude variability functions [5]. The approach creates a methodological foundation for comprehensive analysis of morphological and rhythmic ECS features, enabling improved automated cardiac rhythm disorder diagnosis.

**Mathematical Models Temporal Rhythm Function.** The discrete mathematical model of temporal rhythm function considering extreme amplitude values of ECS characteristic waves is:

$$T_{A_k}(m) = t_{A_k}(m) - t_{A_k}(m - 1), \quad k \in \{P, Q, R, S, T\}, \quad m \in Z \quad (1)$$

where  $t_{A_k}(m)$  is the time moment of reaching the peak of  $k$ -type wave in the  $m$ -th cardiocycle [6].

Statistical parameters include:

- Mathematical expectation estimate:

$$\hat{m}_{T_{A_k}} = \frac{1}{M} \sum_{m=1}^M T_{A_k}(m) \quad (2)$$

- Variance estimate:

$$\hat{d}_{T_{A_k}} = \frac{1}{M-1} \sum_{m=1}^M [T_{A_k}(m) - \hat{m}_{T_{A_k}}]^2 \quad (3)$$

### Amplitude Variability Model

The amplitude variability function for ECS waves is defined as:

$$V_k(m) = A_k(m) - A_k(m-1), \quad k \in \{P, Q, R, S, T\}, \quad m \in \mathbf{Z}, \quad (4)$$

where  $A_k(m)$  is the amplitude of  $k$ -type wave in the  $m$ -th cardiocycle (mV).

Statistical processing include:

- Mathematical expectation:

$$\hat{m}_{V_k} = \frac{1}{M} \sum_{m=1}^M V_k(m) \quad (5)$$

- Variance:

$$\hat{d}_{V_k} = \frac{1}{M-1} \sum_{m=1}^M [V_k(m) - \hat{m}_{V_k}]^2 \quad (6)$$

**Experimental Results.** Experimental validation was conducted on ECS from patients with conditional norm and extrasystole diagnoses. Figure 1 presents the ECS statistical characteristics: mathematical expectation estimate and variance estimate for both patient groups.

Statistical characteristics of temporal rhythm function indicators and amplitude variability function of ECS patient with diagnosis conditional norm and extrasystole shown in Table 1.

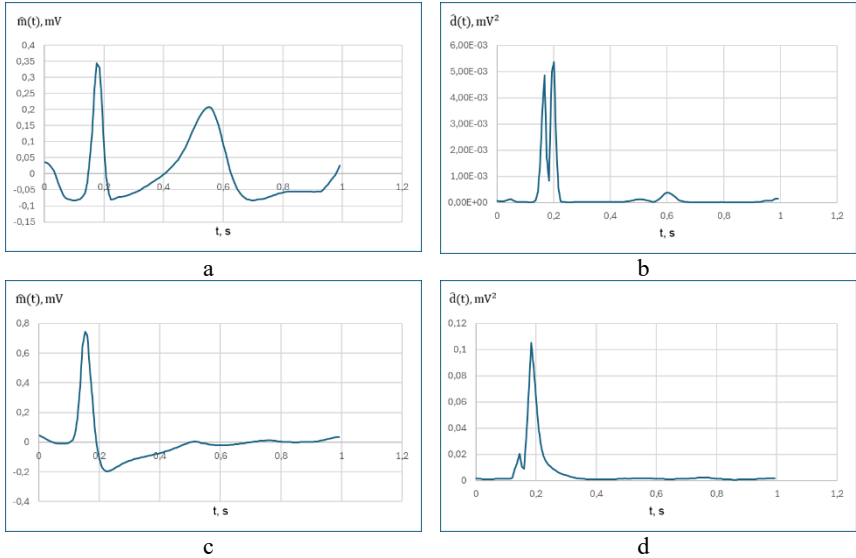


Fig. 1. ECS statistical characteristics: mathematical expectation estimate (a), variance estimate (b) (conditional norm); mathematical expectation estimate (c), variance estimate (d) (extrasystole).

**Table 1.** Statistical characteristics comparison.

Parameter	Conditional Norm	Extrasystole
<i>Temporal Rhythm Function</i>		
$\hat{m}_{T_{A_k}} (s)$	0.776	0.503–0.504
$\hat{d}_{T_{A_k}} (s^2)$	0.00003–0.00004	0.011–0.012
<i>Amplitude Variability Function</i>		
$\hat{m}_{V_k} (mV)$	0.00003–0.00064	0.070–0.452
$\hat{d}_{V_k} (mV^2)$	0.00010–0.00022	78.44–719.20

**Discussion.** Analysis reveals significant differences between normal and pathological ECS characteristics:

1. Temporal stability: patients with conditional norm demonstrate high temporal stability (mathematical expectation 0.776 s, minimal variance 0.00003–0.00004  $s^2$ ). Extrasystole patients show 35% decrease in mathematical expectation and three-order magnitude increase in variance.

2. Amplitude variability: most informative for pathology detection. In extrasystole:

- Mathematical expectation increases 233–15,067 times
- Variance increases by 5–6 orders of magnitude

3. Wave-specific changes: largest amplitude variability changes observed in P and R waves, reflecting disruption of atrial and ventricular depolarization processes.

**Conclusions.** The proposed discrete mathematical models successfully combine temporal rhythm functions with amplitude variability analysis, providing comprehensive ECS assessment. Key findings include:

- High sensitivity to pathological cardiovascular changes
- Amplitude variability as a powerful diagnostic feature
- Potential for integration with machine learning algorithms.

The approach expands methodological foundations for automated cardiac disorder diagnosis, with amplitude variability demonstrating exceptional discriminative power between normal and pathological states. Future work will expand validation to additional arrhythmia types and optimize model parameters for real-time clinical applications.

1. Lyon A., Mincholé A., Martínez J. P., Laguna P., Rodríguez B. *Computational techniques for ECG analysis and interpretation in light of their contribution to medical advances // J. R. Soc. Interface.* – 2018. – 15(138). – № 20170821. <https://doi.org/10.1098/rsif.2017.0821>
2. Madona P., Basti R.I., Zain M.M. *PQRST wave detection on ECG signals // Gac. Sanit.* – 2021. – 35. – P. S364–S369. <https://doi.org/10.1016/j.gaceta.2021.10.052>
3. Lupenko S. *The mathematical model of cyclic signals in dynamic systems as a cyclically correlated random process // Mathematics.* – 2022. – 10(18). – № 3406. <https://doi.org/10.3390/math10183406>
4. Kim Y., Lee M., Yoon J., Kim Y., Min H., Cho H., Park J., Shin T. *Predicting future incidences of cardiac arrhythmias using discrete heartbeats from normal sinus rhythm ECG signals via deep learning methods // Diagnostics.* – 2023. – 13(17). – № 2849. <https://doi.org/10.3390/diagnostics13172849>
5. Sverstiuk A., Mosiy L. *Mathematical modeling of electrocardiogram signal amplitude variability for information technology analysis of their morphological and rhythmic characteristics // Computer-integrated technologies: education, science, production.* – 2025. – No 59. – P. 228–240. <https://doi.org/10.36910/6775-2524-0560-2025-59-29>
6. Mosiy L., Sverstiuk A. *Information technology for electrocardiographic signal analysis based on mathematical models of temporal and amplitude variability // Computer systems and information technologies.* – 2025. – No. 2. – P. 36–44. <https://doi.org/10.31891/csit-2025-2-4>