FREE TRANSVERSE VIBRATIONS OF A LOAD-BEARING ROPE WITH SPRING-COMPLIANT SUPPORTS AT DIFFERENT LEVELS

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This paper investigates the free transverse vibrations of a load-bearing rope with spring-compliant supports at different levels. Using Lagrange's equation and the geometrical approach to flexible string modeling, a nonlinear differential equation was derived to describe the dynamic behavior of the system. Analytical expressions for the reduced mass coefficient were obtained, enabling evaluation of vibration frequencies and amplitudes depending on the rope parameters, load weight, and support compliance. The results demonstrate that spring-compliant supports significantly affect the dynamic response of the system, leading to frequency shifts and increased oscillation amplitudes. The developed methodology can be applied in the design of cableways, suspension bridges, crane systems, and other rope-supported structures.

Keywords: load-bearing rope, transverse vibrations, spring-compliant supports, dynamic loads, reduced mass coefficient, rope-load system

Introduction. Load-bearing ropes are widely used in modern mechanical engineering, transport systems, and construction. They operate under complex conditions and are subjected to both static and significant dynamic loads, which affect their reliability and durability. One of the key research tasks is the investigation of free transverse vibrations of ropes, since these processes define operational stability and safety. Special attention is paid to the influence of spring-compliant supports, which are characterized by high compliance and significantly alter the dynamic behavior of the system [1].

Methods. To study the vibrations, Lagrange's equation was applied with consideration of the geometrical side of the flexible string problem. A single-span rope-load system with spring-compliant supports at different levels was analyzed. Expressions for kinetic and potential energy were derived, considering both the distributed mass of the rope and concentrated loads. The concept of reduced mass was introduced, enabling evaluation of the system's dynamic response at mid-span.

The kinetic energy of the system is expressed as:

$$K = \frac{M(\dot{y}_\partial)^2}{2},$$

where M is the reduced mass of the system and $\dot{y}$ is the velocity of the reduced mass.

Results and discussion. The nonlinear differential equation of free transverse vibrations of the rope (Fig.1) with spring-compliant supports was derived. It was shown that the reduced mass depends on the load weight, the rope inclination angle, and rope tension. The coefficient of reduced mass to the mid-span was obtained analytically: $k_{np} = 1 + \frac{K_{on} A \cdot cos^3 \beta}{l}$.

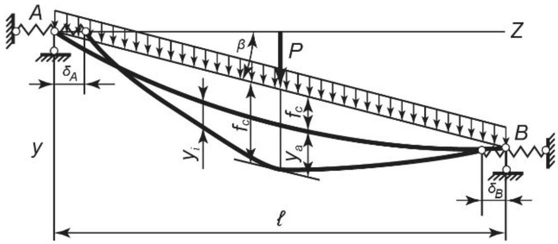

Fig. 1. Schematic deformation of a load-bearing rope with spring-compliant supports at different levels (schematic placeholder).

The equation of free transverse vibrations of a supporting rope with elastic supports at different levels was obtained:

$$\ddot{y}_\partial + k^2 y_\partial = a - \mu(3 f_0 y_\partial^2 + y_\partial^3).$$

It was demonstrated that support compliance reduces the longitudinal dynamic stiffness of the system. This leads to frequency shifts and an increase in vibration amplitudes, which in practice increases the risk of dangerous dynamic forces. The proposed analytical relations can be applied in the design of cableways, crane systems, suspension bridges, and other load-bearing rope structures.

Conclusions. A methodology for determining the reduced mass of a load-bearing rope with spring-compliant supports was developed using Lagrange’s equations and flexible string geometry. A nonlinear differential equation of free transverse vibrations was obtained, describing the dynamic behavior of the rope-load-support system. Analytical dependencies for reduced mass and stiffness were derived, showing their relation to rope parameters, load weight, and support compliance. It was proven that increased compliance of supports significantly modifies vibration frequencies and amplitudes, which must be considered in engineering design.

1. *Kaplun P., Sliepko T. Free transverse vibrations of a load-bearing rope with spring-compliant supports at different levels // Herald of Khmelnytskyi National University. Technical Sciences. – 2025. – 347. – P. 548–553.* https://doi.org/10.31891/2307-5732-2025-347-75