THEORETICAL ANALYSIS OF THE MEAN SQUARE ESTIMATOR ERRORS OF BASIC FREQUENCY

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Functional for estimation of the basic frequency for the mean and covariance functions of periodically non-stationary random processes, grounded on the reduced LS functional, analyzed. Such analysis is provided using solutions of the nonlinear equations with appropriate conditions for maximum existence. The solutions are obtained using the small parameter method.

Keywords: periodically non-stationary random processes, mean square estimator, basic frequency

Introduction. Periodically non-stationary random processes (PNRPs) are the stochastic models of hidden periodicities. Such models can be applied to describe the irregularity and recurrence is time series, which explains different natural processes. Discovering the hidden periodicities using the model of signal as PNRP has been considered in a number of works [1–3]. The statistical properties of the estimator of the non-stationarity period were not analyzed in these investigations, but were briefly characterized in [4, 5]. As shown in [1], the biases and the variances of the LS estimators for the mean and covariance functions and the component estimators formed on the basis of their cyclic statistics quickly converge as the realization length draws.

Investigation of the basic frequency estimation bias. In [6, 7] it was proved, that the LS estimation of the basic frequency for the PNRP mean function can be reduced to finding the maximum point of the following squared functional:

$$\hat{F}_1(\omega) = \frac{1}{2T}\int_{-T}^{T} \hat{m}^2(\omega, t)dt, \tag{1}$$

where

$$\hat{m}(\omega,t) = \hat{m}_0(\omega) + \sum_{k=1}^{L}\left[\hat{m}_k^c(\omega)\cos k\omega t + \hat{m}_k^s(\omega)\sin k\omega t\right], \tag{2}$$

and L is the number of chosen harmonics for searching, $2T$ is the signal realization duration in time. Substitute the component estimator for the mean function (2) into the functional (1) and to provide the time averaging of result, we will obtain approximate solution:

$$\hat{F}_1(\omega) = \hat{m}_0^2 + \frac{1}{2}\sum_{k=1}^{L}[[\hat{m}_k^c(\omega)]^2 + [\hat{m}_k^s(\omega)]^2] +$$

$$+\sum_{\substack{k,l=1\\k\neq l}}^{L}[\hat{m}_k^c(\omega)\hat{m}_l^s(\omega)+\hat{m}_k^s(\omega)\hat{m}_l^c(\omega)]J_0[(k-l)\omega,T]+$$

$$+\sum_{k,l=1}^{L}\left[\hat{m}_k^c(\omega)\hat{m}_l^c(\omega)-\hat{m}_k^s(\omega)\hat{m}_l^s(\omega)\right]J_0\left[(k+l)\omega,T\right] \qquad (3)$$

where $J_0(\omega,T)=\frac{sin\omega T}{\omega T}$. Terms, that depend on the functions $J_0(\omega,T)$, vanish as T increases can be neglected, because of they are the higher order smallness in comparison first two terms in (3). Let us consider the properties of the maximum point of the functional in (3). This point can be found as a solution to the following nonlinear equation:

$$\sum_{k=1}^{L}\left[\hat{m}_k^c(\omega)\frac{d\hat{m}_k^c(\omega)}{d\omega}+\hat{m}_k^s(\omega)\frac{d\hat{m}_k^s(\omega)}{d\omega}\right]=0 .$$

Conclusion. It was shown that LS estimation of the basic frequency for mean and covariance functions of the PNRP can be reduced to searching for the maximum points of the simplified functional, which are defined by the sums of the powers of the harmonics of the test frequency and its multiples.

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