ON CONSIDERATION OF TRANSMISSION CONDITIONS FOR A MULTILAYERED POROELASTIC RECTANGLE WITH A THIN INTERFACE

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The transmission conditions for a multilayered rectangular domain with a thin interface are derived with the help of asymptotic method. The problem is solved by the use of integral transform method, matrix differential calculation apparatus and recurrent correspondences. The numerical investigation of the stress and pore pressure in the rectangular domain is done.

Keywords: poroelastic layered rectangle, thin interface, transmission conditions

The poroelastic rectangular domain that consists of two main parts $\Omega_- = \{0 < x < a - h_0/2, 0 < y < 1\}, \Omega_+ = \{a + h_0/2 < x < a*, 0 < y < 1\}$ and a thin interface $\Omega_0 = \{a - h_0/2 < x < a + h_0/2, 0 < y < 1\}$ is considered. It is supposed that $h_0 \ll a$.

The displacement and pore pressure functions $u^\pm, v^\pm, w^\pm, p^\pm$ in $\Omega_\pm$ and u^0, v^0, w^0, p^0 in Ω_0 satisfy the system of equilibrium and storage equations [1]. The ideal contact conditions are given between Ω_- and Ω_0, Ω_0 and Ω_+ [2]

$$u^\pm\Big|_{x=a\pm h_0/2} = u^0\Big|_{x=a\pm h_0/2}, v^\pm\Big|_{x=a\pm h_0/2} = v^0\Big|_{x=a\pm h_0/2},$$

$$\sigma_x^\pm\Big|_{x=a\pm h_0/2} = \sigma_x^0\Big|_{x=a\pm h_0/2}, \tau_{xy}^\pm\Big|_{x=a\pm h_0/2} = \tau_{xy}^0\Big|_{x=a\pm h_0/2},$$

$$p^\pm\Big|_{x=a\pm h_0/2} = p^0\Big|_{x=a\pm h_0/2}, k_\pm \frac{\partial p^\pm}{\partial x}\bigg|_{x=a\pm h_0/2} = k_0 \frac{\partial p^0}{\partial x}\bigg|_{x=a\pm h_0/2}$$

The sides $y = 0, y = 1$ are impermeable under slide contact conditions

$$v\Big|_{y=0,1} = 0, \tau_{xy}\Big|_{y=0,1} = 0, \frac{\partial p}{\partial y}\bigg|_{y=0,1} = 0$$

The right side $x = a*$ is fixed and permeable

$$u^+\Big|_{x=a_*} = 0, v^+\Big|_{x=a_*} = 0, p^+\Big|_{x=a_*} = 0,$$

the left side $x = 0$ is loaded by both mechanical and fluid pressure loads

$$\sigma_x^-\Big|_{x=0} = -L(y) - \alpha P(y), \tau_{xy}^-\Big|_{x=0} = T(y), p^-\Big|_{x=0} = P(y)$$

In order to find transmission conditions between main parts Ω_- and Ω_+ the asymptotic method [3] is applied. According to it the assumptions about the poroelastic constants $G_0, \kappa_0, \alpha_0, k_0, S_{p0}$ of the interface Ω_0 should be given. Muskhelishvli's and Biot's constants κ_0, α_0 correspondingly change in the fixed intervals $1 < \kappa_0 < 3, 0 < \alpha_0 \le 1$, so they are always of the same order as corresponding constants on the main parts. The assumptions about the orders of the shear modulus G_0, permeability coefficient k_0 and storativity of the pore space S_{p0} give 8 different cases:

1) G_0, k_0, S_{p0} are of the same order as corresponding constants of the main parts $G_\pm, k_\pm, S_{p\pm}$. In this case displacements, stress, pore pressure and normal flux are continuous through the interface.
2) $G_0 \ll G_\pm$. In this case there are jumps of the displacements, all other functions are continuous

$$u_0^+\Big|_{x=a+0} - u_0^-\Big|_{x=a-0} = \frac{h_0}{G_0}\frac{\kappa_0 - 1}{\kappa_0 + 1}\sigma_{x,0}^-\Big|_{x=a-0},$$

$$v_0^+\Big|_{x=a+0} - v_0^-\Big|_{x=a-0} = \frac{h_0}{G_0}\tau_{xy,0}^-\Big|_{x=a-0}$$

3) $k_0 \ll k_\pm$. In this case there is a jump of the pore pressure, all other functions are continuous

$$p_0^+\Big|_{x=a+0} - p_0^-\Big|_{x=a-0} = \frac{h_0}{k_0}k_-\frac{\partial p_0^-}{\partial x}\Bigg|_{x=a-0}$$

4) $G_0 \gg G_\pm$. In this case there are jumps of the stress, all other functions are continuous

$$\sigma_{x,0}^+\Big|_{x=a+0} - \sigma_{x,0}^-\Big|_{x=a-0} = -G_0 h_0 \frac{\partial^2 u_0^-}{\partial y^2}\Bigg|_{x=a-0},$$

$$\tau_{xy,0}^+\Big|_{x=a+0} - \tau_{xy,0}^-\Big|_{x=a-0} - G_0 h_0 \frac{\kappa_0 + 1}{\kappa_0 - 1}\frac{\partial^2 v_0^-}{\partial y^2}\Bigg|_{x=a-0}$$

5) $k_0 \gg k_\pm$. In this case there is a jump of the normal flux, all other functions are continuous

$$k_+ \frac{\partial p_0^+}{\partial x}\bigg|_{x=a+0} - k_- \frac{\partial p_0^-}{\partial x}\bigg|_{x=a-0} = -k_0 h_0 \frac{\partial^2 p_0^-}{\partial y^2}\bigg|_{x=a-0}$$

6) $S_{p0} \gg S_{p\pm}$. In this case there is a jump of the normal flux, all other functions are continuous

$$k_+ \frac{\partial p_0^+}{\partial x}\bigg|_{x=a+0} - k_- \frac{\partial p_0^-}{\partial x}\bigg|_{x=a-0} = h_0 S_{p0}\, p_0^-\big|_{x=a-0}$$

7) $G_0 \ll G_\pm, k_0 \ll k_\pm$. In this case there are jumps of the displacements, normal stress, pore pressure and normal flux, tangential stress functions are continuous

$$u_0^+\big|_{x=a+0} - u_0^-\big|_{x=a-0} = \left[\sigma_{x,0}^- \frac{(\kappa_0 - 1)\varpi}{\alpha_0 G(\kappa_0 + 1)} + k_- \frac{\partial p_0^-}{\partial x}\frac{2\vartheta}{\alpha_0}\right]\bigg|_{x=a-0},$$

$$v_0^+\big|_{x=a+0} - v_0^-\big|_{x=a-0} = \frac{h_0}{G_0}\tau_{xy,0}^-\big|_{x=a-0},$$

$$p_0^+\big|_{x=a+0} - p_0^-\big|_{x=a-0} = \left[\sigma_{x,0}^- \frac{2\vartheta}{\alpha_0} + k_- \frac{\partial p_0^-}{\partial x}\frac{\varpi}{\alpha_0 k}\right]\bigg|_{x=a-0},$$

$$\sigma_{x,0}^+\big|_{x=a+0} - \sigma_{x,0}^-\big|_{x=a-0} = \left[\sigma_{x,0}^- 2\vartheta + k_- \frac{\partial p_0^-}{\partial x}\frac{G_0\varpi}{k_0 G}\right]\bigg|_{x=a-0}$$

$$k_+ \frac{\partial p_0^+}{\partial x}\bigg|_{x=a+0} - k_- \frac{\partial p_0^-}{\partial x}\bigg|_{x=a-0} = \left[\sigma_{x,0}^- \frac{k_0(\kappa_0 - 1)\varpi}{G_0 k(\kappa_0 + 1)} + k_- \frac{\partial p_0^-}{\partial x} 2\vartheta\right]\bigg|_{x=a-0},$$

where $\vartheta = \sinh^2\left(\frac{\alpha_0 W h_0}{2\varepsilon}\right), \varpi = \frac{\sinh\left(\frac{\alpha_0 W h_0}{\varepsilon}\right)}{W}, W = \sqrt{\frac{(\kappa_0 - 1)}{Gk(\kappa_0 + 1)}}$.

8) $G_0 \ll G_\pm, k_0 \ll k_\pm, S_{p0} \gg S_{p\pm}$. In this case there are jumps of the displacements, and pore pressure all other functions are continuous

$$u_0^+\big|_{x=a+0} - u_0^-\big|_{x=a-0} = \frac{h_0}{G_0}\frac{S_p(\kappa_0 - 1)}{(\kappa_0 + 1)}\sigma_{x,0}^-\big|_{x=a-0},$$

$$v_0^+\big|_{x=a+0} - v_0^-\big|_{x=a-0} = \frac{h_0}{G_0}\tau_{xy,0}^-\big|_{x=a-0}$$

These 8 cases provide different transmission conditions, all other combinations of the poroelastic constants' orders can be derived by combining these 8 basis cases. Note also that S_{p0} plays significant role only when it is large. The cases when S_{p0} is of the same order as $S_{p\pm}$ or is less than this value give the same transmission conditions.

The original problem is reduced to the one-dimensional boundary value problem with the help of finite sin-, cos- Fourier transform [4], and it is formulated in a vector form. The matrix differential calculation apparatus [5] is applied to construct the general solution of the one-dimensional boundary value problem, and unknown constants are found with the help of recurrent correspondences [5] based on the derived transmission conditions.

The analytical solutions for the problem with different transmission conditions are constructed. Using them the investigation of stress and pore pressure inside the two-layered rectangular domain is provided.

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